Differential Eqns and Linear Alg.

Dr Taylor Daniels

- Look at Brightspace
- Look at Course page, Pearson, Syllabus
- Look at graphing tools.
- Pre-reqs: Calc III; (recall basics of vectors); Complex numbers supplemental note; basics of physics ($x, v, a, F = ma$)

(Ordinary) Differential Equation (ODE):

- single independent variable (x, t, etc.)
- Single output function and its derivatives,

  $y(x), y'(x), y''(x), \ldots$

  or $x(t), \frac{dx}{dt}, \frac{d^2x}{dt^2}, \ldots$

Big ideas: Writing down formulas for models is hard/impossible.

- Easier to describe "laws" governing changes of the system.
- Then, get initial data, "integrate" the changes, $\rightarrow$ obtain model.

Newton's Law of Cooling:

$$\frac{dT}{dt} = k(A - T)$$

A = Ambient temp.

t = time

$T(t)$ = temperature of object. (over time)

k (>0) is some "constant of proportionality"

Ex (verifying sols) **ODE** $y'' + \pi^2 y = 0$ ①

"candidate function" $y(x) = \cos(\pi x)$

Does this $y(x)$ satisfy the ODE?

$$y'(x) = -\pi \sin(\pi x)$$
$$y''(x) = -\pi^2 \cos(\pi x).$$

So

$$y''(x) + \pi^2 y(x) = -\pi^2 \cos(\pi x) + \pi^2 \cos(\pi x) = 0,$$

so $y(x) = \cos(\pi x)$ is a solution to the ODE ①

Ex: (verifying sols) ODE: $\{ y' = 12x^2(y^2+1) \}$ ②

Does $y(x) = \tan(4x^3 + C)$ satisfy this ODE?

(C is some unknown constant)

$$y' = \sec^2(4x^3+C)\cdot(12x^2) \qquad (\tan^2 x + 1 = \sec^2 x)$$
$$= (\tan^2(4x^3+C)+1)\cdot(12x^2)$$
$$y' = (y^2+1)\cdot(12x^2).$$

So yes, $y(x) = \tan(4x^3 + C)$ satisfies the ODE / is a solution of the ODE.

For example, if $\tilde{y}(x) = e^x$, then

$$\tilde{y}' = e^x = \tilde{y} \neq 12x^2(\tilde{y}^2 + 1),$$

so $\tilde{y} = e^x$ is not a solution of ②

Ex (cont.) What is a solution / choice of C that makes $y(x) = \tan(4x^3 + C)$ satisfy the initial condition $y(0) = 1$?

$$1 = y(0) = \tan(0 + c) = \tan(c),$$

$$\Longrightarrow \quad \tan(c) = 1,$$

so $c = \pi/4$ works.

In total, the function

$$y(x) = \tan\left(4x^3 + \frac{\pi}{4}\right)$$

is a solution to the <u>initial value problem</u> (IVP)

$$\left\{ \begin{array}{l} y' = 12x^2(y^2+1) \\ y(0) = 1 \end{array} \right\}$$